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MULTIPLE TRANSMISSION
LINE DISCONTINUITIES

by

David A. Snyder

A Thesis

Presented to the Graduate Committee

of Lehigh University

in Candidacy for the Degree of

Master of Science

in

Electrical Engineering

Lehigh University

1981

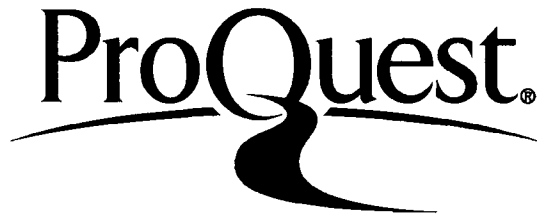
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/ Chairman of Department

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ABSTRACT

The effect of multiple splices in an electromagnetic transmission line is investigated. The splicing operation considered creates an abrupt increase in the outer conductor diameter of a coaxial transmission line. Analysis of the resulting overall reflection coefficient is presented.

Multiple splicing will increase the reflection. A straightforward computer program calculates the reflection coefficient with arbitrary splice spacings. The case of uniform spacing is solved analytically and the results are used to check the computer program in a specific case. The analysis is conducted in terms of scattering parameters instead of the usual impedance matrix. One distinct advantage of the approach is that the effect of line attenuation can be readily incorporated.

In order to obtain experimental verification a test cable was cut and spliced at 39 points. A technique for measuring the reflection using the Computer Operated Transmission Measuring Set is developed and an accuracy analysis performed. Observed reflections are 5 dB larger than calculated. This is due to the fact that the conductor geometry in the actual splice does not accurately conform to the assumed theoretical configuration.

If the reflection coefficient of each individual splice is adjusted 5 dB, there is good agreement between experiment, analysis and computer simulation. Therefore all pertinent effects are considered, and the program and splice model may be confidently applied to future design of cable systems.

Introduction

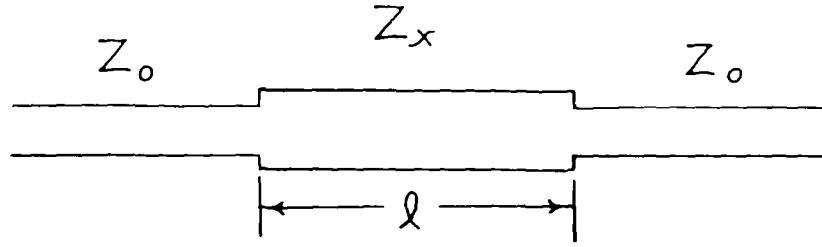
Electromagnetic wave transmission lines ideally have a constant and uniform cross section. Any local deviation from the nominal parameters and geometry will reflect some portion of the propagating signal. While the reflection from one such discontinuity may be small, if there are many periodically spaced discontinuities then for certain frequencies the reflections will add constructively and the overall reflection coefficient will be large. Use of such a periodically disturbed transmission line as a communication channel could result in serious degradation of the transmitted signal. Consequently it becomes important to be able to characterize the individual discontinuity and compute the overall reflection coefficient.

There are many situations resulting in multiple discontinuities in a transmission line. Variations in the transmission line's geometry or dielectric might inadvertently occur during cable manufacture or might even be inherent in the cable design. Another possibility is that the transmission line is periodically tapped to connect transmitters and receivers, e.g., the mother board of a microcomputer. Finally system design might simply require that the cable be cut and then spliced back together, often in some quasi uniform pattern. This example of multiple imperfect splices in a transmission line forms the basis for the subsequent analysis and provides experimental verification of the results.

1. Individual Splice Physical Model

Consider a spliced transmission line, where the splice is made as nearly identical to the uniform line as possible except for the conductor spacing. The spacing change causes the line's local characteristic impedance to vary with axial position, $Z_c(x)$. Such impedance variation might also result from a change in the dielectric permittivity.

Suppose that this splice causes the line's impedance to abruptly change from its uniform value Z_0 , to some value Z_x characteristic of the splice. At a short distance l away there is an abrupt return to the uniform line value.



The reflection from this discontinuity may be approximated by using the theory of small reflection. Each junction is modeled as ideal and only first order reflections are considered. By symmetrically placing the reference plane a phase factor is eliminated from the final result.

$$\begin{aligned}
 R &= \left(\frac{Z_x - Z_o}{Z_x + Z_o} \right) e^{jk l} + \left(\frac{Z_o - Z_x}{Z_o + Z_x} \right) e^{-jk l} \\
 &= \left(\frac{Z - 1}{Z + 1} \right) 2j \sin k l
 \end{aligned}$$

Z is the normalized impedance Z_x/Z_o , and k is the propagation constant of the TEM mode considered

$$k = 2\pi f \sqrt{\epsilon}/c$$

with c the speed of light, f frequency and ϵ dielectric constant.

At low frequency (wavelength much longer than splice length) the splice becomes a point discontinuity with reflection

$$R \sim jk l / 2 \left(\frac{Z - 1}{Z + 1} \right), \quad k l \ll 1 \quad (1.1)$$

The validity of this approximation is demonstrated by the result of Appendix 1. Here the effects of junction capacitance and interaction are included by using the technique of variational

calculus. At frequencies below waveguide cutoff the junction reactance is negligible and the reflection is asymptotic to

$$R \sim jkl \frac{1}{2} \left[Z + \frac{1}{Z} \right], \quad kL \ll 1 \quad (1.2)$$

This same equation is obtained using conventional transmission line theory, Fig. 1.

The essential equivalence of equations 1.1 and 1.2 is illustrated in Fig. 2a by plotting the functional dependence of the reflection against Z .

Both analyses predict a reflection magnitude $\rho = |R|$, that is directly proportional to frequency. This is confirmed in Fig. 2b which plots measurements made on a single splice of the construction indicated in Fig. 3. Reflection coefficients are obtained from the impedance bridge by measuring the impedance of spliced (Z_s) and unspliced (Z_u) short cable lengths with identical terminations. The reflection is then computed as

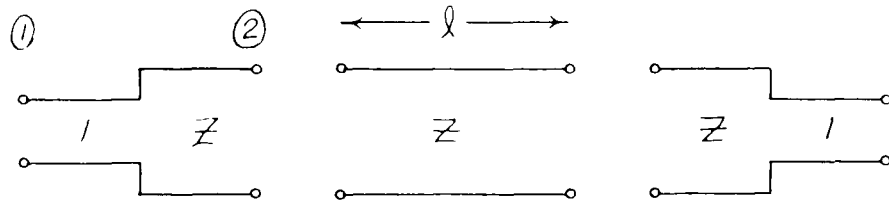
$$R = \frac{Z_s - Z_u}{Z_s + Z_u}$$

The pulse echo test set and time domain reflectometer measure the reflection magnitude directly but the results must be converted to the frequency domain. The frequency associated with a raised cosine pulse of duration τ is determined by periodically extending the waveform, i.e., $f=1/2\tau$. For a time domain reflectometer with rise time t_r the corresponding frequency is $f=1/2\pi t_r$ obtained by matching the slope of a sine wave at zero crossing with the slope of the step input. The 5 dB discrepancy between theory and experiment is discussed in Section 6.

Since the splice discontinuity is assumed to be lossless and symmetric then knowledge of the reflection R determines the transmission T , ref. 1, as

$$|T| = \sqrt{1 - |R|^2} = \sqrt{1 - \rho^2}$$

$$\angle T = \angle R - \frac{\pi}{2} = 0$$



$$[S] = \begin{bmatrix} \frac{z-1}{z+1} & \frac{2}{z+1} \\ \frac{2z}{z+1} & \frac{1-z}{1+z} \end{bmatrix}, \begin{bmatrix} 0 & e^{-jkl} \\ e^{jkl} & 0 \end{bmatrix}, \begin{bmatrix} \frac{1-z}{1+z} & \frac{2z}{z+1} \\ \frac{2}{z+1} & \frac{z-1}{z+1} \end{bmatrix}$$

$$[a] = \frac{1}{2z} \begin{bmatrix} z+1 & z-1 \\ z-1 & z+1 \end{bmatrix}, \begin{bmatrix} e^{jkl} & 0 \\ 0 & e^{-jkl} \end{bmatrix}, \frac{1}{2} \begin{bmatrix} z+1 & 1-z \\ 1-z & z+1 \end{bmatrix}$$

$$[a]_{\text{total}} = \frac{1}{4z} \begin{bmatrix} e^{jkl}(1+z)^2 - e^{-jkl}(1-z)^2 & (e^{jkl} - e^{-jkl})(1-z^2) \\ (e^{jkl} - e^{-jkl})(z^2-1) & e^{jkl}(1+z)^2 - e^{-jkl}(1-z)^2 \end{bmatrix}$$

$$S_{11} \text{ total} = \frac{a_{21}}{a_{11}} = \frac{2j \sin kl}{\left(\frac{1+z}{1-z}\right)e^{jkl} - \left(\frac{1-z}{1+z}\right)e^{-jkl}} \sim jkl \frac{1}{2} \left(\frac{1}{z} - z\right) \quad kl \ll 1$$

Figure 1.

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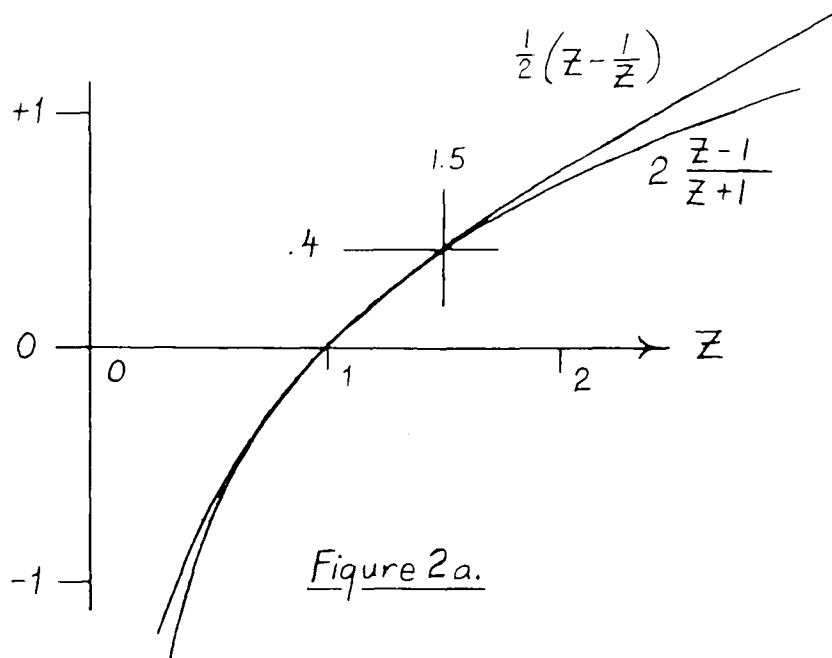


Figure 2a.

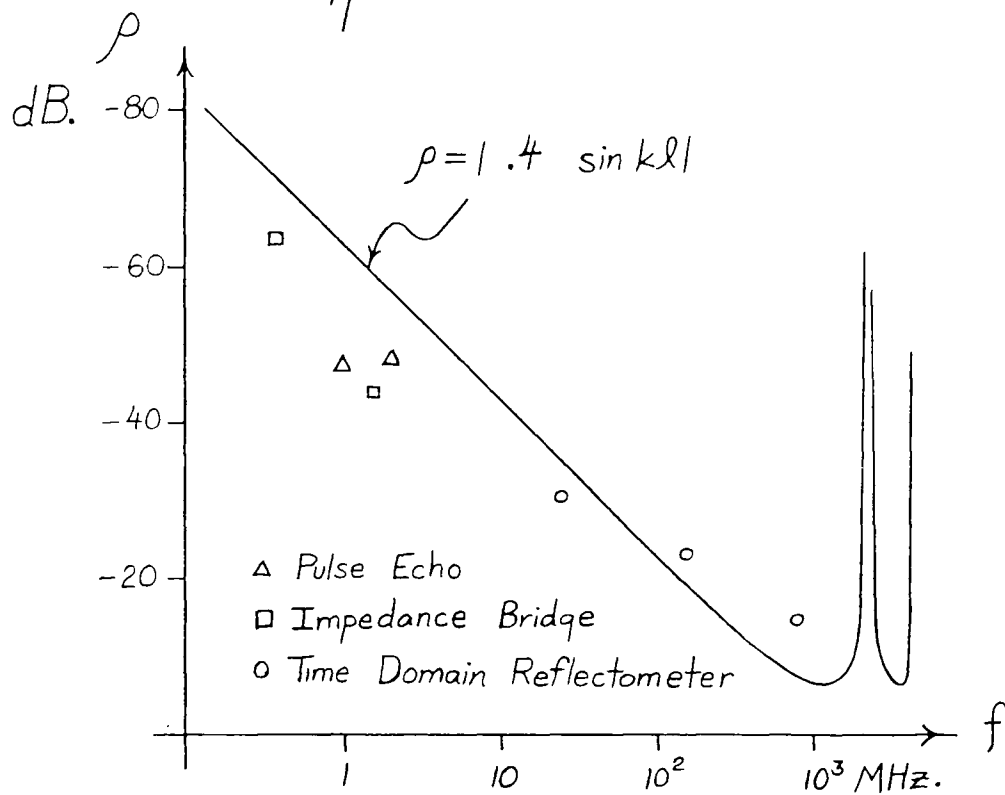
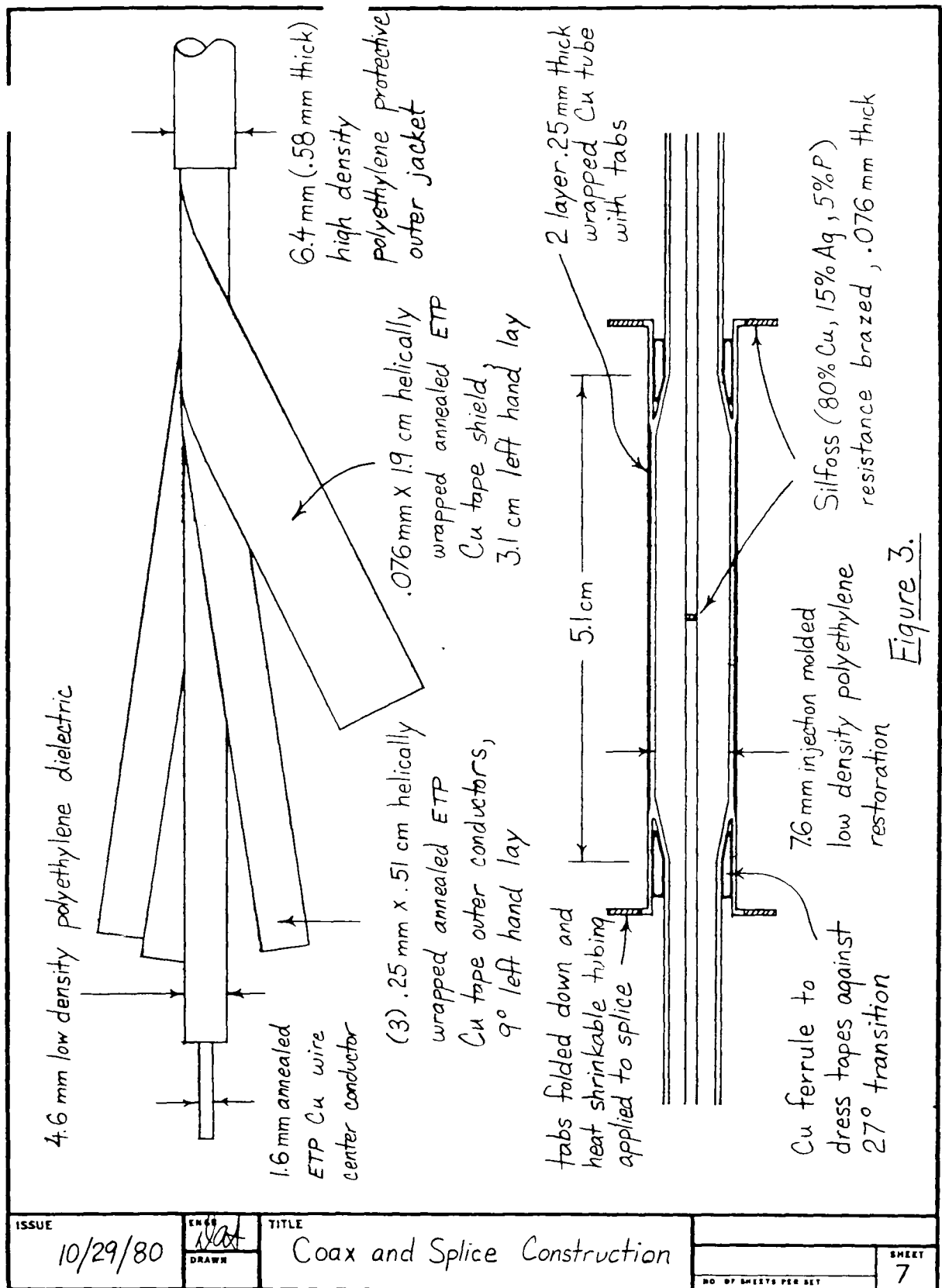


Figure 2b.

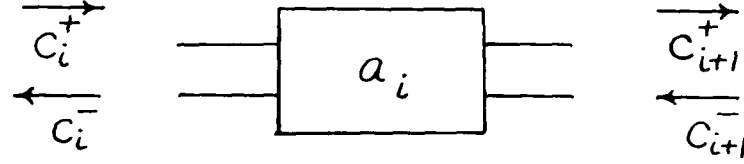
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R and T are just the scattering parameters $s_{11} = s_{22}$ and $s_{12} = s_{21}$ respectively of the two port network equivalent to the discontinuity. Multiple discontinuities are then modeled as a cascade of such networks

2. Arbitrary Discontinuity Spacing

The reflection from and transmission thru a transmission line with multiple discontinuities are obtained by multiplying together the transmission matrices representing the individual elements. These matrices are defined in terms of the forward and backward traveling waves at each port



$$\begin{bmatrix} c_i^+ \\ c_i^- \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} c_{i+1}^+ \\ c_{i+1}^- \end{bmatrix} \quad (2.1)$$

$$[a] = \begin{bmatrix} \frac{1}{s_{21}} & -\frac{s_{22}}{s_{21}} \\ \frac{s_{11}}{s_{21}} & s_{12} - \frac{s_{11}s_{22}}{s_{21}} \end{bmatrix} \quad (2.2)$$

For a reciprocal 2 port $s_{12} = s_{21}$ giving the well known result that $\det[a] = 1$. Also for a symmetric element $s_{11} = s_{22}$ and the matrix $[a]$ is antisymmetric, $a_{12} = -a_{21}$.

For the uniform line with propagation constant $\alpha + jk$ and length d , the transmission matrix is

$$[a_u]_i = \begin{bmatrix} e^{(\alpha + jk)d} & 0 \\ 0 & e^{-(\alpha + jk)d} \end{bmatrix}$$

while for the lossless symmetric point discontinuity it is

$$[a_x] = \frac{1}{\sqrt{1 - \rho^2}} \begin{bmatrix} 1 & -j\rho \\ j\rho & 1 \end{bmatrix}$$

Consider a communication trunk comprised of N uniform lines of length d_i each followed by a discontinuity. The total transmission matrix of the complete trunk is given by

$$[a_t] = \prod_{i=1}^N [a_u]_i [a_x]$$

Fig. 4 shows the block diagram for a computer program to perform this matrix multiplication. It was used to generate the plots of Figs. 7 and 11. This program is useful for determining if specific discontinuity patterns will produce unacceptable magnitudes of reflection with consequent distortion of the transmitted signal.

The case of uniform spacing is amenable to direct analysis and the results were used to check the operation of the computer program.

3. Uniform Discontinuity Spacing

When the discontinuities are uniformly spaced the line may be broken into N identical unit cells. Without loss of generality choose the reference planes halfway between discontinuities so that the cells are symmetric. By neglecting attenuation the unit cell is lossless and requires only 2 parameters to describe it; the phase of the reflection coefficient

$$\angle s_{11} = kd + \frac{\pi}{2}$$

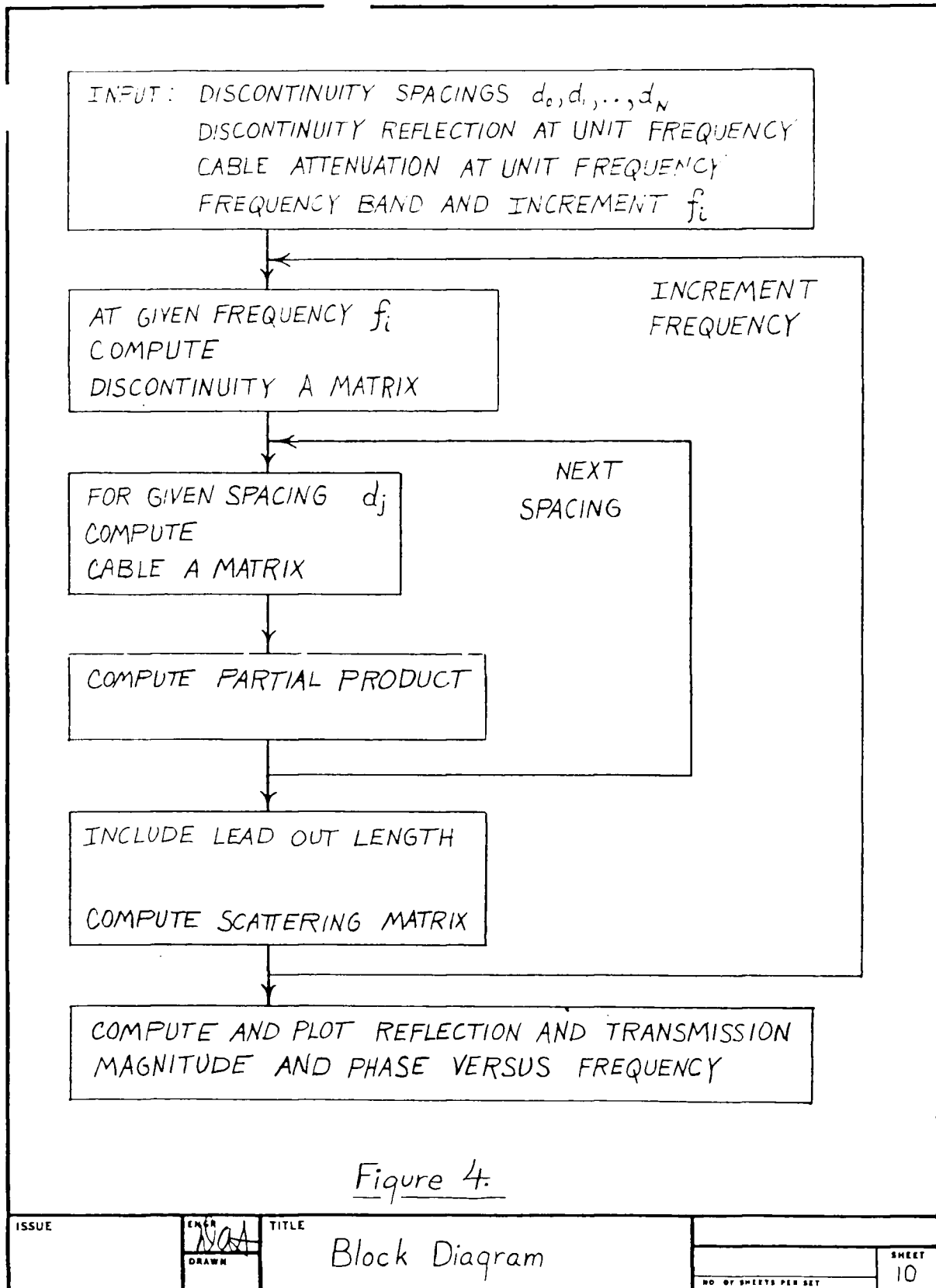


Figure 4.

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where d is the length of the cell, and the magnitude of the reflection coefficient:

$$|s_{11}| = \rho$$

The remaining scattering matrix element s_{21} is obtained from

$$\begin{aligned} |s_{21}| &= |s_{11}| = \frac{\pi}{2} + kd \\ |s_{21}|^2 &= 1 - |s_{11}|^2 = 1 - \rho^2 \end{aligned}$$

To determine the reflection from a finite number of discontinuities consider first the propagation characteristics of an infinite sequence of unit cells. The wave amplitudes, c , are observable only at the reference planes, if a propagating wave exists it will be of the form

$$\begin{aligned} c_{i+1}^+ &= c_i^+ e^{-\gamma d} \\ c_{i+1}^- &= c_i^- e^{-\gamma d} \end{aligned}$$

Substitute this into eq. 2.1 to obtain

$$\begin{bmatrix} a_{11} - e^{\gamma d} & a_{12} \\ a_{21} & a_{22} - e^{\gamma d} \end{bmatrix} \begin{bmatrix} c_{i+1}^+ \\ c_{i+1}^- \end{bmatrix} = 0$$

which has a nontrivial solution if the determinant equals zero

$$e^{2\gamma d} - (a_{11} + a_{22}) e^{\gamma d} + 1 = 0 \quad (3.1)$$

or

$$\cosh \gamma d = (a_{11} + a_{22})/2$$

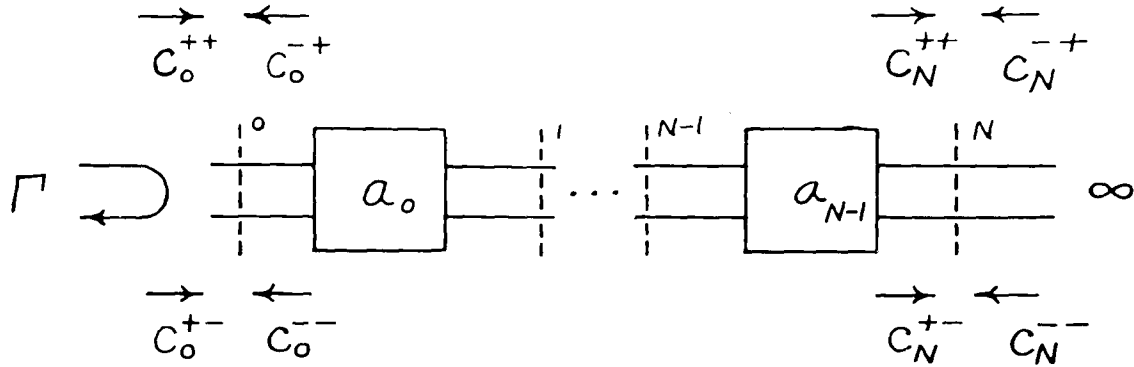
There are two solutions for γd , positive and negative, each has a corresponding eigenvector composed of forward and backward traveling normal transmission line waves in the ratio

$$\Gamma_B^+ \equiv \frac{c_i^{+-}}{c_i^{++}} = \frac{e^{\gamma d} - a_{11}}{a_{12}} = \frac{a_{21}}{e^{\gamma d} - a_{22}} \quad (3.2)$$

or

$$\Gamma_B^- \equiv \frac{c_i^{+-}}{c_i^{--}} = \frac{a_{12}}{e^{-\gamma d} - a_{11}} = \frac{e^{-\gamma d} - a_{22}}{a_{21}}$$

Γ_B is the Bloch reflection coefficient for positive or negative traveling Bloch waves. The second superscript for c_i will be needed to compute the reflection from a finite number of cells



The reflection observed at the input terminals is

$$\begin{aligned} \Gamma &= \frac{c_0^{-+} + c_0^{--}}{c_0^{++} + c_0^{+-}} \\ &= \frac{c_N^{-+} e^{\gamma Nd} + c_N^{--} e^{-\gamma Nd}}{c_N^{++} e^{\gamma Nd} + c_N^{+-} e^{-\gamma Nd}} \end{aligned} \quad (3.3)$$

With the output terminated into the characteristic impedance of the uniform line there will be no negative traveling wave at plane N, therefore

$$c_N^{-+} + c_N^{--} = 0$$

and

$$\frac{c_N^{++}}{c_N^{--}} = \frac{c_N^{++}}{c_N^{-+}} \frac{c_N^{-+}}{c_N^{--}} = -\frac{1}{\Gamma_B^+}$$

Substituting into eq. 3.3

$$\Gamma = \frac{e^{\gamma Nd} - e^{-\gamma Nd}}{\frac{e^{\gamma Nd}}{\Gamma_B^+} - \Gamma_B^- e^{-\gamma Nd}}$$

For the case of symmetric unit cells $\Gamma_B^- = \Gamma_B^+ \equiv \Gamma_B$ and the above becomes

$$\Gamma = \frac{e^{j\gamma d} - \Gamma_R e^{-j\gamma d}}{\Gamma_R e^{j\gamma d} - e^{-j\gamma d}} \quad (3.4)$$

This equation will be evaluated for the case of lossless symmetric discontinuities in a lossless line. The reflection from the splice alone is

$$S_{11,\text{splice}} = j\rho$$

For the symmetric unit cell consisting of the splice and two lossless lines of length $d/2$

$$S_{11,\text{cell}} = e^{-jkd} j\rho$$

and then

$$S_{21,\text{cell}} = e^{-jkd} \sqrt{1 - \rho^2}$$

From the relation 2.2 between transmission and scattering matrices

$$\frac{a_{11} + a_{22}}{2} = \frac{\cos kd}{\sqrt{1 - \rho^2}}$$

and

$$\frac{a_{11} - a_{22}}{2} = j \frac{\sin kd}{\sqrt{1 - \rho^2}}$$

The eigenvalue equation 3.1 may be rewritten as

$$\begin{aligned} e^{\pm \gamma d} &= \frac{a_{11} + a_{22}}{2} \pm \sqrt{\left(\frac{a_{11} + a_{22}}{2}\right)^2 - 1} \\ e^{\pm \gamma d} &= \frac{1}{\sqrt{1 - \rho^2}} \left(\cos kd \pm \sqrt{\rho^2 - \sin^2 kd} \right) \end{aligned} \quad (3.5)$$

This expression describes the propagation of the wave along the infinite periodic structure. To facilitate its use in evaluating eq. 3.4 approximations will be made for several special cases.

When $\rho^2 < \sin^2 kd$ then $e^{\gamma d}$ is complex with magnitude equal one. The wave propagates freely, consequently these regions are called passbands. The propagation constant γ must be pure imaginary, say $\gamma = j\beta$, with

$$\tan \beta d = \sqrt{1 - \frac{\rho^2}{\sin^2 kd}} + \tan kc \quad (3.6)$$

This equation is the dispersion relation within the passbands. For the situations where $\rho^2 \ll \sin^2 kd$ the effect of the splices is a negligible shift of the wave's phase. This approximation is valid over most of the passband if the length of a splice, l , is much less than the spacing between splices, d .

As the excitation frequency increases eventually $\rho^2 \geq \sin^2 kd$. $e^{\gamma d}$ is now pure real with magnitude other than one. The wave is attenuated, not from energy absorption (the unit cells are lossless), but rather from coupling to the reverse traveling mode. These regions, known as stopbands, are approximately centered about $kd = m\pi$, m an integer. Expanding the sinusoids in eq. 3.5 around $m\pi$ gives

$$e^{\gamma d} = \sqrt{\frac{1 \pm \rho}{1 \mp \rho}} \left[\pm 1 - \frac{(kd - m\pi)^2}{2\rho} \right], \quad kd \rightarrow m\pi \quad (3.7)$$

where $+$ corresponds to m even and $-$ to m odd.

Any one of the equations 3.2 is suitable for evaluating Γ_B , for instance

$$\Gamma_B = \frac{a_{21}}{\frac{a_{11} - a_{22}}{2} + \sqrt{\left(\frac{a_{11} + a_{22}}{2}\right)^2 - 1}}$$

Now

$$a_{21} = j\rho/\sqrt{1 - \rho^2}$$

then

$$\Gamma_B = \frac{j\rho}{j \sin kd + \sqrt{\rho^2 - \sin^2 kd}} \quad (3.8)$$

In the passbands Γ_B is real with

$$\Gamma_B \sim \frac{\rho}{2 \sin kd}, \quad \rho^2 \ll \sin^2 kd \quad (3.9)$$

In the stopbands Γ_B is complex with magnitude equal, one. At the m th band center:

$$\Gamma_B = \Gamma_0 e^{-jkd} = m\pi$$

while near the center

$$\Gamma_B \sim \frac{\pi}{2} \mp \frac{m\pi + kd}{\rho}, \quad kd = m\pi$$

The behavior of $e^{\gamma d}$ and Γ_B is best illustrated by the polar plots of Fig. 5a and b.

Eq. 3.4 may now be evaluated for each case. In the passbands away from the edges $|\Gamma_B| \ll 1$, and the second denominator term may be neglected with the result

$$\Gamma \sim \Gamma_B \left(1 - e^{-2j\beta Nd} \right) \quad (3.11)$$

then

$$|\Gamma| \sim \Gamma_B |2 \sin \beta Nd|, \quad |\Gamma_B| \ll 1$$

So the reflection magnitude varies from 0 when $\beta Nd = q\pi$, q an integer, to a maximum when

$\beta Nd = \left(q - \frac{1}{2} \right) \pi$ (assuming Γ_B does not vary too rapidly). If the latter condition is substituted

into the exact reflection, eq. 3.4, then the locus of the maxima is obtained

$$|\Gamma|_{\max} = \frac{2}{\frac{1}{\Gamma_B} + \Gamma_B} = \left| \frac{\rho}{\sin kd} \right| \quad (3.12)$$

The frequencies for maximum or minimum reflection are determined by substituting the appropriate condition into the dispersion relation. When Γ_B is small the q th maximum is approximately located at

$$k_q \sim \frac{\pi}{Nd} \left(q - \frac{1}{2} \right), \quad |\Gamma_B| \ll 1$$

This expression is not appropriate near the band edges.

Actually the reflection magnitude may be expressed as an exact function of β by taking the magnitude of eq. 3.4 and using

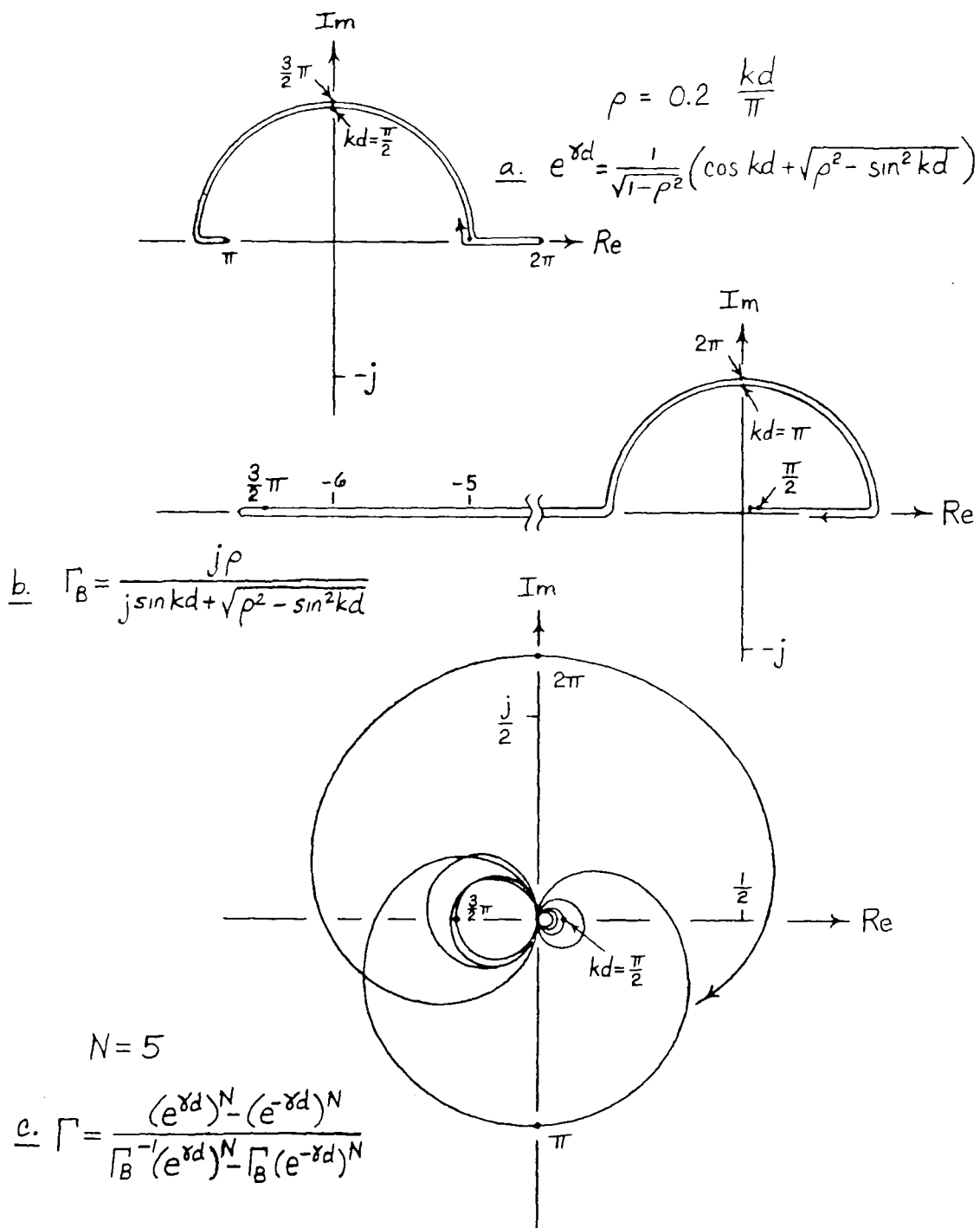


Figure 5.

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| ISSUE | ENGR | TITLE |
| 11/25/80 | DRAWN | Polar Plots of $e^{\gamma_d}, \Gamma_B, \Gamma$ |
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$$\Gamma_B = \Gamma_B - \frac{2}{j} \sqrt{\sin^2 kd - \rho^2} = \frac{2}{j} \sqrt{1 - \rho^2 \sin^2 \beta d}$$

to obtain

$$|\Gamma|_{\text{pass}} = \left[\frac{1 - \rho^2}{\rho^2} \left(\frac{\sin \beta d}{\sin \beta Nd} \right)^2 + 1 \right]^{-\frac{1}{2}} \quad (3.13a)$$

The advantage of this form is that it can be evaluated as the band edge is approached, $\beta d \rightarrow m\pi$

$$|\Gamma|_{\text{edge}} = \left[\frac{1 - \rho^2}{(\rho N)^2} + 1 \right]^{-\frac{1}{2}} \sim N\rho, \quad N\rho \ll 1 \quad (3.13b)$$

The phase of the reflection computed using the approximation 3.11 is

$$\angle \Gamma \sim \frac{\pi}{2} - \beta Nd, \quad |\Gamma_B| \ll 1. \quad (3.14)$$

The reflection appears to be caused by a single discontinuity with phase $\frac{\pi}{2}$ located halfway down the periodic structure (since $\beta \approx k$).

The reflection at the center of the m th stopband is (using 3.7 and 3.10 with $kd = m\pi$ in 3.4)

$$\Gamma = \pm j \frac{1 - \left(\frac{1 - \rho}{1 + \rho} \right)^N}{1 + \left(\frac{1 - \rho}{1 + \rho} \right)^N} \sim \pm j N\rho, \quad N\rho \ll 1 \quad (3.15)$$

where the approximation is valid only when the condition on $N\rho$ is satisfied. This result is intuitive in that it is the worst case situation where all the reflections add in phase.

Finally if eqs. 3.7 and 3.10 are substituted into 3.4, the result expanded for small ρ , the magnitude and phase of the reflection computed, and only the most significant term in $kd - m\pi$ retained, then in the stopband

$$\begin{aligned} \Gamma^+ &\sim N_p \left(1 - \frac{1}{2} \frac{(1+m\pi+kd)}{p} \right)^{\frac{1}{2}}, \quad kd \rightarrow m\pi \\ \Gamma^- &\sim 1 - \frac{\pi}{2} + N(m\pi+kd), \quad kd \rightarrow m\pi \end{aligned} \quad (3.16)$$

Here again $+$, $-$ refers to m even, odd

Eqs. 3.11 to 3.16 determine the expected reflection coefficient for uniformly spaced splices. This function $\Gamma(kd)$ is illustrated by Nyquist plot of Fig. 5c for the case $N = 5$ and

$$\pi \frac{l}{d} 2 \frac{Z-1}{Z+1} = .2.$$

4. Test Cable Construction and Estimated Reflection

A coaxial transmission line was cut and spliced to provide a test cable for experimental verification of the analysis.

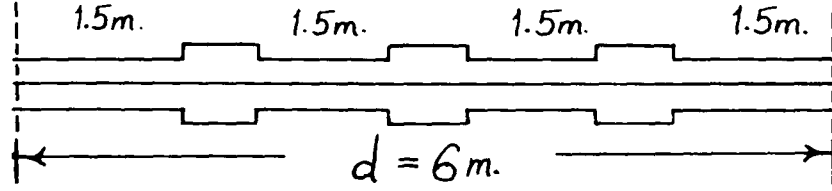
Details of the coax construction and splicing technique may be found in Fig. 3. Basically the coax consists of a .163 cm. diameter copper inner conductor, a copper outer conductor with .457 cm. inside diameter, and a low density polyethylene dielectric. The nominal high frequency impedance is 41Ω with an attenuation of 5 dB/km at 1 MHz. The splice is made by butt brazing the center conductor, injection molding polyethylene over the exposed wire, and restoring the outer conductor by brazing in a wrapped tube of copper. The molded dielectric is 5.08 cm. long by .762 cm. diameter, consequently the outer conductor diameter changes significantly at the splice. The inner conductor diameter is constant and the brazing of both inner and outer conductors is of such quality that the resistance introduced is negligible.

The transition in outer conductor diameter is a 27° taper which is modeled as an abrupt change. The normalized characteristic impedance for the splice is readily calculated from the diameter change;

$$Z = \frac{\ln \frac{.762}{.163}}{\ln \frac{.457}{.163}} = 1.5$$

where the polyethylene's dielectric constant is assumed to be the same in splice and cable.

The test cable overall length is 79m and contains 39 splices. Instead of uniform spacing the splices were separated 1.5, 1.5, 3, 1.5, 1.5, 3, ... m. By eliminating every fourth splice in an otherwise uniform arrangement the stopband frequencies are reduced by a factor of four facilitating measurements with available equipment. The unit cell now becomes



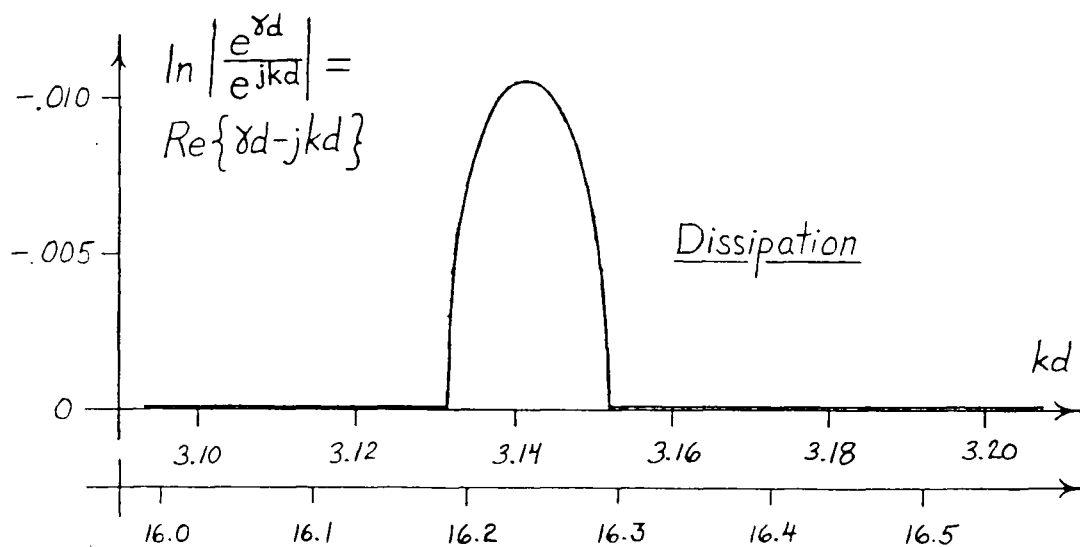
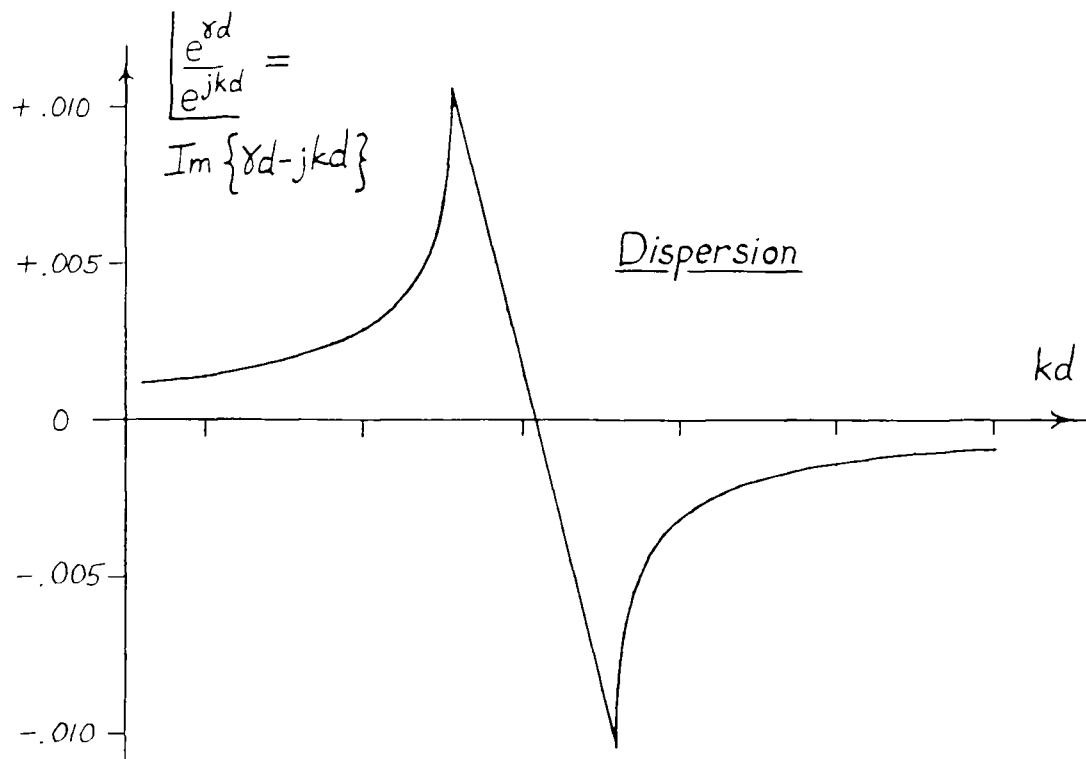
The reflection coefficient may be computed from the theory for small reflections. Since the cell is symmetric the calculation is more convenient when referenced to the center.

$$\begin{aligned}
 R &= \frac{Z-1}{Z+1} \left[\exp jk \left(\frac{d}{4} + \frac{l}{2} \right) - \exp jk \left(\frac{d}{4} - \frac{l}{2} \right) \right. \\
 &\quad \left. + \exp jk \frac{l}{2} - \exp jk \left(-\frac{l}{2} \right) \right. \\
 &\quad \left. + \exp jk \left(-\frac{d}{4} + \frac{l}{2} \right) - \exp jk \left(-\frac{d}{4} - \frac{l}{2} \right) \right] \\
 R &= j2 \frac{Z-1}{Z+1} \sin kl \left[1 + 2 \cos \frac{kd}{2} \right] \\
 \rho &\sim j2 \frac{Z-1}{Z+1} kl \left[1 + 2 \cos \frac{kd}{2} \right], \quad kl \ll 1 \quad (4.1)
 \end{aligned}$$

This equation for ρ is used in the dispersion relation 3.6 and the dissipation equation 3.7 to generate $k - \gamma$ diagram of Fig. 6. The expected magnitude of the reflection from 13 such unit cells is determined from equations 3.6, 3.13, and 3.15. Fig. 7 plots structure return loss versus frequency where

$$SRL = -20 \log |\Gamma|$$

This predicted behavior may now be compared with measurements made on the test cable.



$Z = 1.5$

Figure 6.

MHz. for test cable
 $d = 6.1 \text{ m}$, $c = 1.98 \cdot 10^8 \text{ m/s}$.

| | | | | |
|-------------------|-------------------|-----------------------------------------|-----------------------|-------------|
| ISSUE 10/29/80 | ENG <i>Mat</i> | TITLE Calculated k- γ Diagram | NO. OF SHEETS PER SET | SHEET 20 |
| | DRAWN | | | |

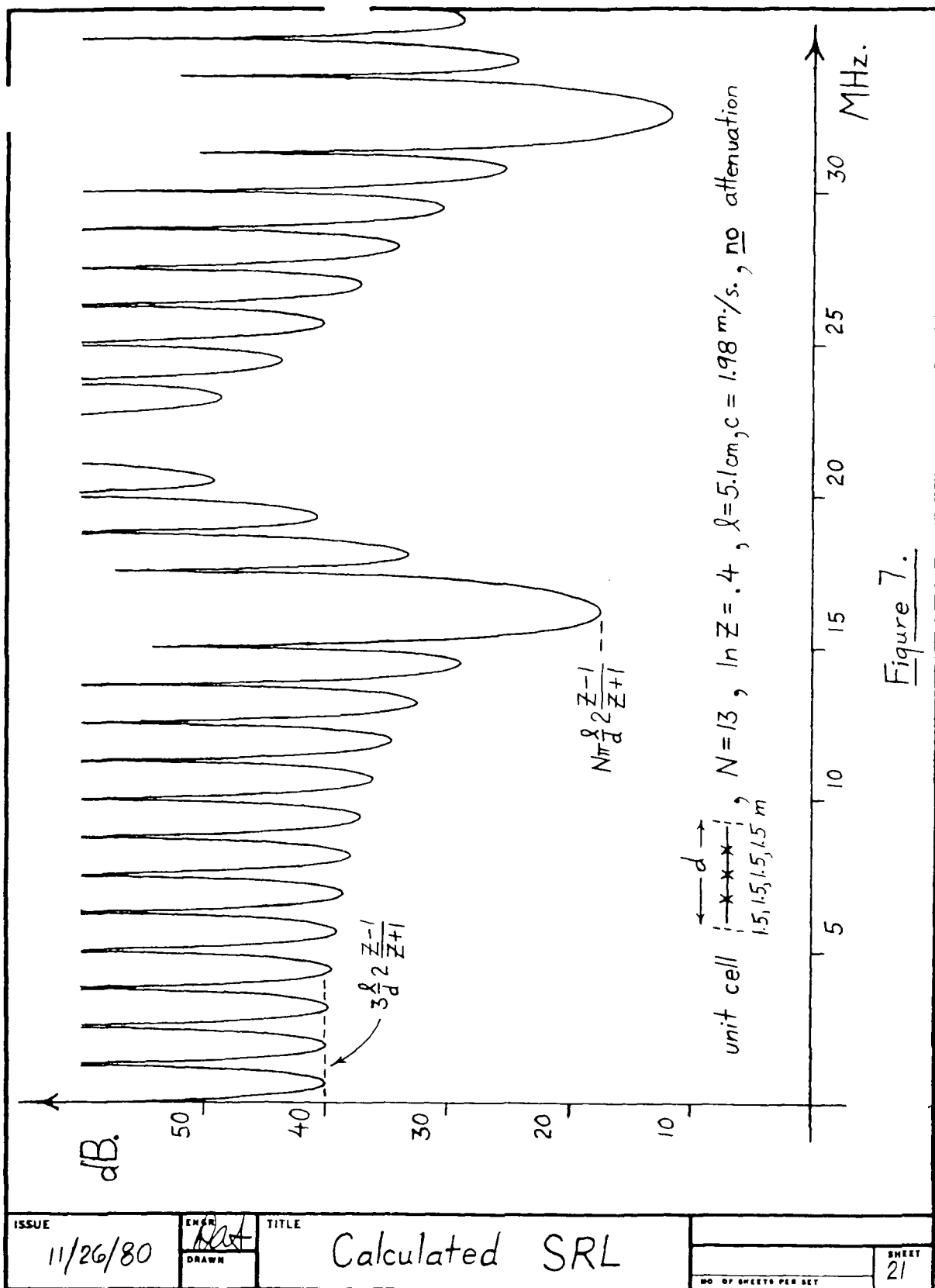


Figure 7.

5. Theory of Measurement

The plot of Fig. 7 applies when the test cable is terminated into the characteristic impedance of the uniform line. Typically measurement sets have input impedances of 50 or 75 Ω . The reflection from the mismatch to a 41 Ω line would obscure the effect under consideration. Furthermore at low frequencies the line impedance increases to 44 Ω so that simple resistive matching pads and terminations are inadequate. Rather than design frequency dependent networks to achieve the required accuracy one can employ a computational technique to eliminate mistermination errors.

The Computer Operated Transmission Measuring Set developed by the Bell Telephone Laboratories is preprogrammed to perform the required calculations. The test cable was measured on the COTMS located at Whippany, N. J. A review of the theory, explanation of the procedure applied, and an accuracy analysis follows.

Basically COTMS consists of a frequency synthesizer with output port, and an input port with detector. The complex ratio of the voltages at input and output is determined and saved in memory, this measurement will be denoted by T with appropriate subscripts. To make a return loss measurement the output and input ports are connected to any linear 3 port network. The voltage ratios with the third port terminated into a known reference standard of impedance R (T_s), into an open circuit (T_∞), and into a short circuit (T_o) are determined. Then the measurement (T_x) is made with the unknown impedance X connected. The reflection coefficient referenced to R,

$$\Gamma_{\text{COTMS}} = \frac{X-R}{X+R} \quad (5.1)$$




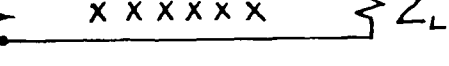
may then be computed, ref. 2 and 3, as

$$\Gamma_{\text{COTMS}} = \frac{(T_\infty - T_o)(T_x - T_s)}{(T_s - T_o)(T_x - T_\infty) + (T_x - T_o)(T_s - T_\infty)}$$

The reflection coefficient obtained is for the unknown terminated into the reference impedance independent of the actual input impedance of the third port of the network

Now the problem here is how to reference measurements on a two port (the test cable) to the impedance of the uniform line. Ideally the reference standard and far end test termination should be infinitely long lines. If instead an imperfectly terminated uniform line of the same length as the test cable is used as the reference, and if this same termination is used on the test cable, then the computed reflection will closely approximate the reflection with ideal terminations.

This procedure is represented schematically as;

| Measurement | Measurand | |
|--------------|-----------------------------------------------------------------------------------------------------------------|------------------|
| T_s | $R_{in} \rightarrow$  Z_L | uniform coax |
| T_o |  | short circuit |
| T_{∞} |  | open circuit |
| T_x | $X_{in} \rightarrow$  Z_L | spliced coax |

where R_{in} , X_{in} are the complex impedances presented to the test set. The question is how accurately does the computed reflection, Γ_{COTMS} , measure the desired reflection, Γ (for splices in a uniform and infinite line). Reflection coefficients are always computed relative to some reference impedance, the impedance used is usually obvious and not mentioned explicitly. However for this accuracy analysis a second subscript is needed to denote the reference impedance used. Then

$$\Gamma_{L,B} = \frac{Z_L - Z_B}{Z_L + Z_B}$$

$$\Gamma_{L,B} = \frac{Z_L - Z_B}{Z_L + Z_B}$$

where Z_B is the Bloch wave impedance. Obviously it is defined by

$$Z_B = \frac{1 + \Gamma_{B,o}}{1 - \Gamma_{B,o}} Z_o, \quad \text{with} \quad \Gamma_{B,o} \equiv \Gamma_B \quad (5.2)$$

Similarly

$$R_{in} = \frac{1 + \Gamma_{in,o}}{1 - \Gamma_{in,o}} Z_o \quad \text{where} \quad \Gamma_{in,o} = e^{-2\gamma_o L} \Gamma_{L,o} \quad (5.3)$$

and

$$X_{in} = \frac{1 + \Gamma_{in,B}}{1 - \Gamma_{in,B}} Z_B \quad \text{where} \quad \Gamma_{in,B} = e^{-2\gamma_B L} \Gamma_{L,B} \quad (5.4)$$

Now $\Gamma_{L,B}$ may be referenced to Z_o as

$$\Gamma_{L,B} = \frac{Z_L - Z_B}{Z_L + Z_B} = \frac{\Gamma_{L,o} - \Gamma_{B,o}}{1 - \Gamma_{L,o} \Gamma_{B,o}} \quad (5.5)$$

by using eq. 5.2 and a similar one for Z_L . By substituting eq. 5.2 and 5.5 into 5.4, then 5.3 and 5.4 into 5.1; canceling Z_o and dropping the second subscript now that all reflections are referenced to the same impedance, Z_o ; and finally factoring out the desired reflection Γ , eq. 3.4; obtain

$$\Gamma_{\text{COTMS}} = \Gamma \frac{1 + \Gamma_L \left\{ \frac{1 - e^{-2\gamma_o L}}{\Gamma} - \left(\Gamma_B + \frac{1}{\Gamma_B} \right) + \Gamma_L e^{-2\gamma_o L} \right\}}{1 - \Gamma_L \left\{ \Gamma \left(1 + e^{-2\gamma_o L} \right) + \Gamma_L e^{-2\gamma_o L} \left[1 - \Gamma \left(\Gamma_B + \frac{1}{\Gamma_B} \right) \right] \right\}}$$

The fraction is the relative error, notice that as the misterrmination reflection, Γ_L , goes to zero the measured reflection becomes just the desired reflection. Now given some mismatch consider several special cases.

In the passband the expected reflection, Γ , goes to zero when $\beta Nd = q\pi + (\gamma_B - \gamma_0)$. Then the first term in the numerator brackets dominates and

$$\Gamma_{\text{COTMS}} \sim \Gamma_L \left[1 - e^{-2\gamma_0 l} \right], \quad \Gamma \rightarrow 0$$

At low frequencies the dispersion relation eq. 3.6 becomes

$$\beta d \sim \left[1 - \frac{1}{2} \left(\frac{\rho}{kd} \right)^2 \right] kd, \quad kd \ll 1$$

so that

$$\Gamma_{\text{COTMS}} \sim j \frac{1}{2} \Gamma_L \left(\frac{\rho}{kd} \right)^2$$

For the test cable it is expected that

$$\frac{\rho}{kd} \sim 3 \left(\frac{l}{d} \right)^2 \frac{Z-1}{Z+1} = .01, \quad kd \ll 1$$

Then

$$\text{SRL} = -20 \log |\Gamma_L| + 86 \text{ dB}$$

demonstrating that the minima will be sharply defined even with significant mismatch. However near the stopbands γ_B and γ_0 will differ significantly and $e^{-2\gamma_0 l}$ may approximately equal -1 instead of $+1$ so that

$$\Gamma_{\text{COTMS}} = 2\Gamma_L$$

Here the load mismatch causes a residual reflection at the expected minima.

A much more important consideration is the error at reflection maxima,

$$\beta Nd = \pi \left[q + \frac{1}{2} \right]$$

Then

$$\Gamma = \frac{2}{\frac{1}{\Gamma_B} + \Gamma_B}$$

which when substituted makes numerator and denominator identical. The procedure introduces *no* error into measurement of the passband maxima

Finally at the center of the m th stopband $2\gamma_0 l = 2jkNd = 2jNm\pi$ so $e^{-2\gamma_0 l} = -1$, also $\Gamma_B = (-1)^m$, therefore

$$\frac{1}{\Gamma_B} + \Gamma_B = 0.$$

The measured reflection becomes

$$\Gamma_{\text{COTMS}} = \Gamma \frac{1 + \Gamma_L^2}{1 - \Gamma_L(2\Gamma + \Gamma_L)} \sim \Gamma(1 + 2\Gamma_L\Gamma), \quad |\Gamma| \gg |\Gamma_L|$$

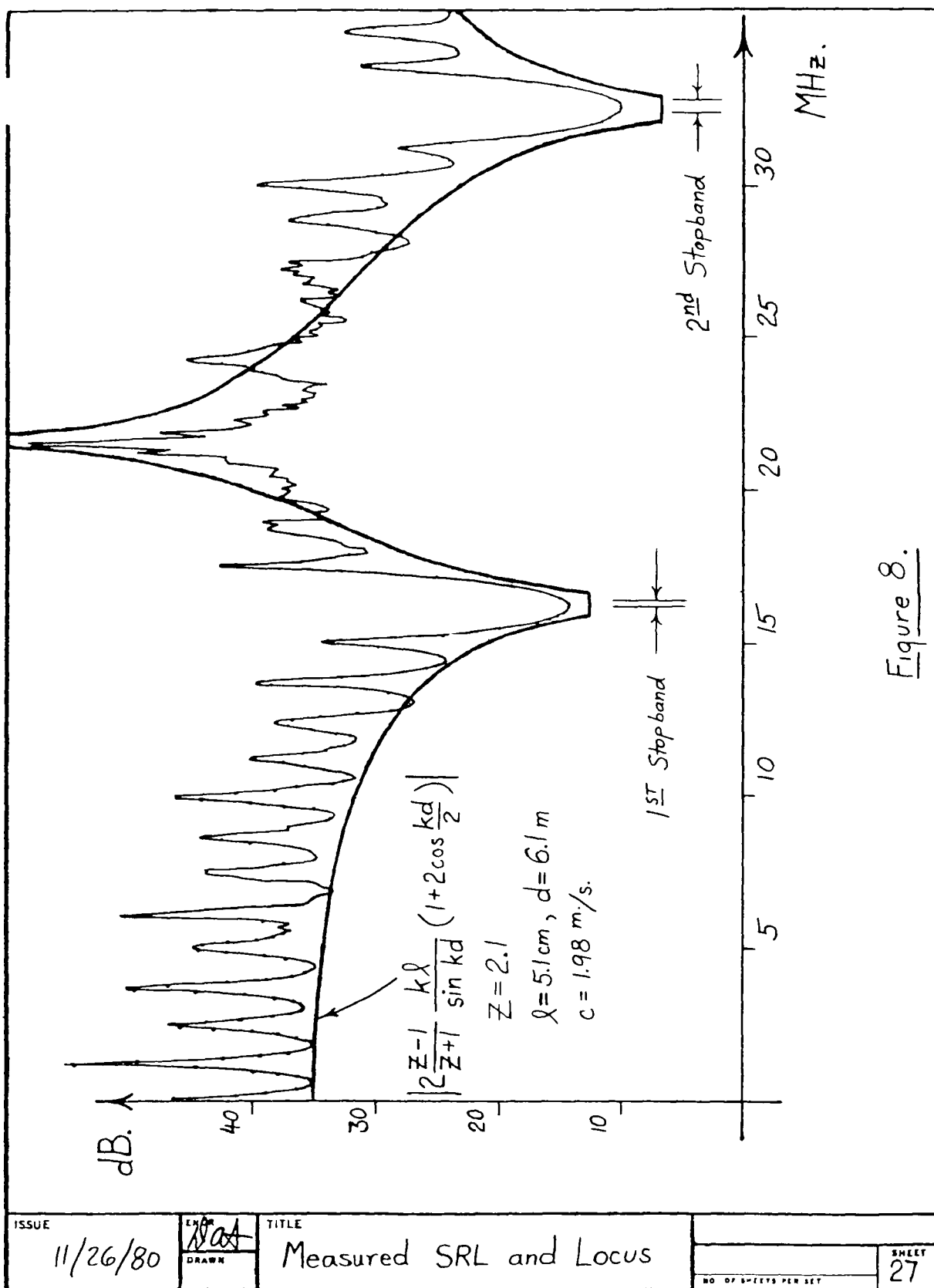
Given a stopband reflection of about -10dB and a termination mismatch of $2\ \Omega$ out of $41\ \Omega$ nominal then the procedure introduced error is only -0.14dB or 1.6%

6. Experimental Results

The above procedure was applied to a test cable comprised of 13 unit cells, where each cell is 6m. long and contains 3 splices. This gives a total of 39 splices and an overall length of 78m. The reference was a uniform cable of the same length $\pm .3\text{m}$. The SRL obtained is plotted in Fig. 8 along with the predicted locus of the maxima. The locus is obtained by substituting eq. 4.1 into 3.12

$$\text{SRL}(\text{locus}) = -20 \log \left| 2 \frac{Z-1}{Z+1} \frac{kl}{\sin kd} \left[1 + 2 \cos \frac{kd}{2} \right] \right| \quad (6.)$$

where the value $Z = 2.1$ was used. Since $2 \frac{Z-1}{Z+1}$ is a multiplicative factor, then changing its value merely shifts the locus up or down on the logarithmic plot. The value 2.1 was chosen to match theory and experiment at low frequency where line attenuation is negligible.



Apparently there is more reflection from the splice than the analysis predicts, $Z = 1.5$. This discrepancy might be attributed to changes in the polyethylene dielectric constant caused by the injection molding operation. The reflection, eq. 1.1, may be recalculated to include such an effect

$$R = jk/2 \frac{Z - \sqrt{\epsilon_n}}{Z + \sqrt{\epsilon_n}}$$

where ϵ_n is the dielectric constant within the splice ϵ_x normalized by the dielectric constant within the uniform cable ϵ

$$\epsilon_n \equiv \epsilon_x / \epsilon$$

Here k and z are the propagation constant and splice characteristic impedance assuming no dielectric constant change. If the geometry effects are adequately modeled and therefore $Z = 1.5$ then the ϵ_n required to account for the discrepancy may be calculated

$$\frac{1.5 - \sqrt{\epsilon_n}}{1.5 + \sqrt{\epsilon_n}} = \frac{2.1 - 1}{2.1 + 1}$$

Knowing that $\epsilon = 2.28$ gives

$$\epsilon_x = 1.16$$

The injection molding process could not alter the polyethylene dielectric constant this drastically.

The probable explanation is the method of outer conductor restoration. To prevent melting of the polyethylene the outer conductor tabs may not be brazed immediately adjacent to the dielectric surface. As a result the outer conductor electrical diameter at the transition is more than the nominal .762 cm. Such an irregularity could cause the observed additional reflection.

In Fig. 8 note the horizontal truncations of the passband maxima locus. These are the stop-band maxima calculated using eq. 3.15. The 2.0 and 3.3 dB discrepancies with experiment are caused by line attenuation. Loss may be accounted for by simply imagining that the reflection

occurs at a point halfway down the cable (see eq. 3.14). Then the magnitude of the reflection is reduced by the round trip loss, which is just the single pass attenuation of the reference cable. Justification for this general rule is provided in Appendix 2.

Fig. 9 plots the COTMS measured insertion loss of the reference cable. The data point scatter at the upper frequencies is caused by secondary reflections from imperfect matching pads. None the less the square root of frequency dependence is evident. At the stopband frequencies, 16.2 and 32.4 MHz the loss is 2.3 and 3.3 dB., in good agreement with the return loss discrepancy.

Also indicated in Fig. 8 are the stopband edges; 16.14, 16.34, 32.29 and 32.68 MHz, determined by solving the equation

$$|\sin kd| = \rho \sim .72 kl, \quad kl \ll 1$$

Having established the magnitude of the reflection from a single splice, consider now the phase which should theoretically be $\pi/2$. First suppose the actual phase, θ , for the individual splice deviates $\Delta\theta$ from the expected value,

$$\theta = \frac{\pi}{2} + \Delta\theta.$$

The s parameters for the unit cell become

$$\begin{aligned} s_{11} &= e^{-j(kd - \Delta\theta)} j\rho \\ s_{21} &= e^{-j(kd - \Delta\theta)} \sqrt{1 - \rho^2} \end{aligned}$$

Therefore all the previously derived relations remain valid if kd is replaced with $kd - \Delta\theta$. In the passbands the dispersion relation for small reflections becomes

$$\beta d = kd - \Delta\theta + o[\rho^2/\sin^2 kd]$$

For the test cable the order of approximation is $o[10^{-4}]$. Consequently by noting the frequencies, $f_q = \frac{c}{2\pi} k_q$, where the reflection is minimum, $\beta Nd = q\pi$, the discontinuity reflection phase is measured by

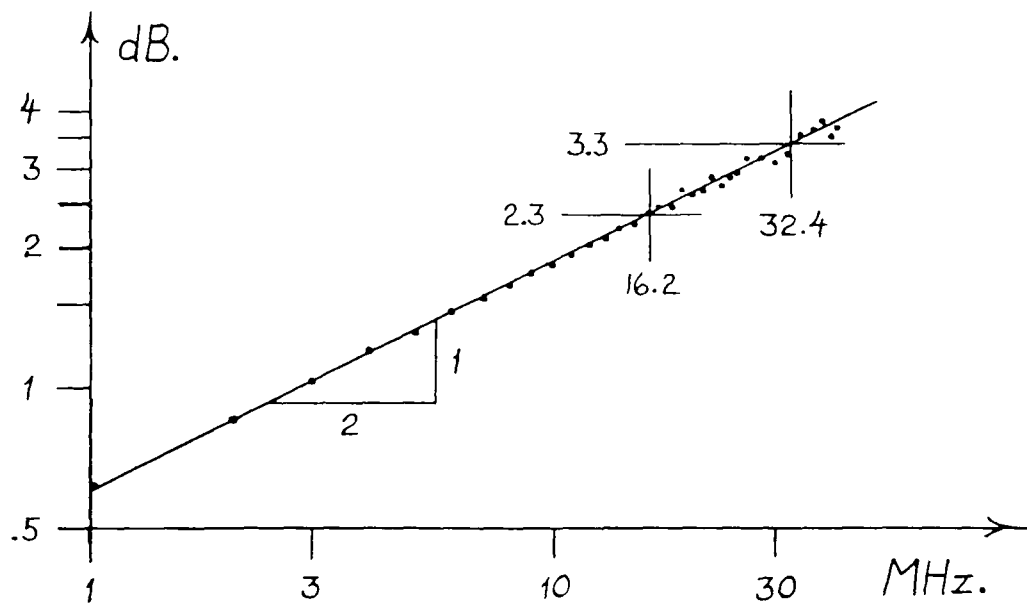


Figure 9a.

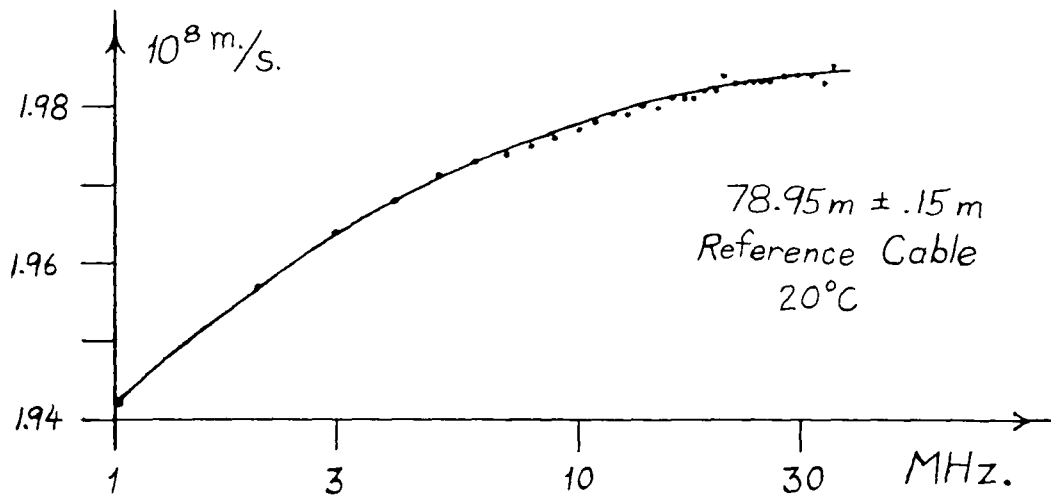


Figure 9b.

| | | | | |
|------------------|---------------------------------|--------------------------------------------------|-----------------------------|-------------|
| ISSUE 8/18/80 | EXHIBIT DRAWN [Signature] | TITLE Cable Attenuation and Phase Velocity | NO. OF SHEETS PER SET 30 | SHEET 30 |
|------------------|---------------------------------|--------------------------------------------------|-----------------------------|-------------|

$$\Delta\theta = k_q d = \frac{q\pi}{\lambda}$$

The return loss plot, Fig. 8, determines, the f_q to within .1 MHz. The computed phase deviation is then accurate to

$$d\Delta\theta = \frac{2\pi d}{c} df = .02 \text{ radians or } 1^\circ.$$

Also the velocity c , Fig. 9b, is uncertain to $\pm .4\%$ because of the length measurement. At 30 MHz the error is

$$d\Delta\theta = -\frac{2\pi fd}{c} \frac{dc}{c} = .02 \text{ radians}$$

Table 1 for the phase deviation shows that the wave reflected from the individual splice has as the expected phase of $90^\circ \pm 1.5^\circ$

Finally the measured phase of the reflection versus frequency is plotted in Fig. 10. The behavior is best visualized using the computer generated Nyquist plot for Γ of Fig. 11. The shift of phase with frequency determines the distance to the apparent source of the reflection. If a discontinuity with constant reflection phase is located the distance D from the reference plane then

$$\begin{aligned} |\Gamma| &= 2kD \\ \frac{\partial |\Gamma|}{\partial f} &= 4\rho \frac{D}{c} \end{aligned}$$

From the slope in Fig. 10

$$D = \frac{c}{4\pi} \frac{\partial |\Gamma|}{\partial f} = 33.5\text{m}$$

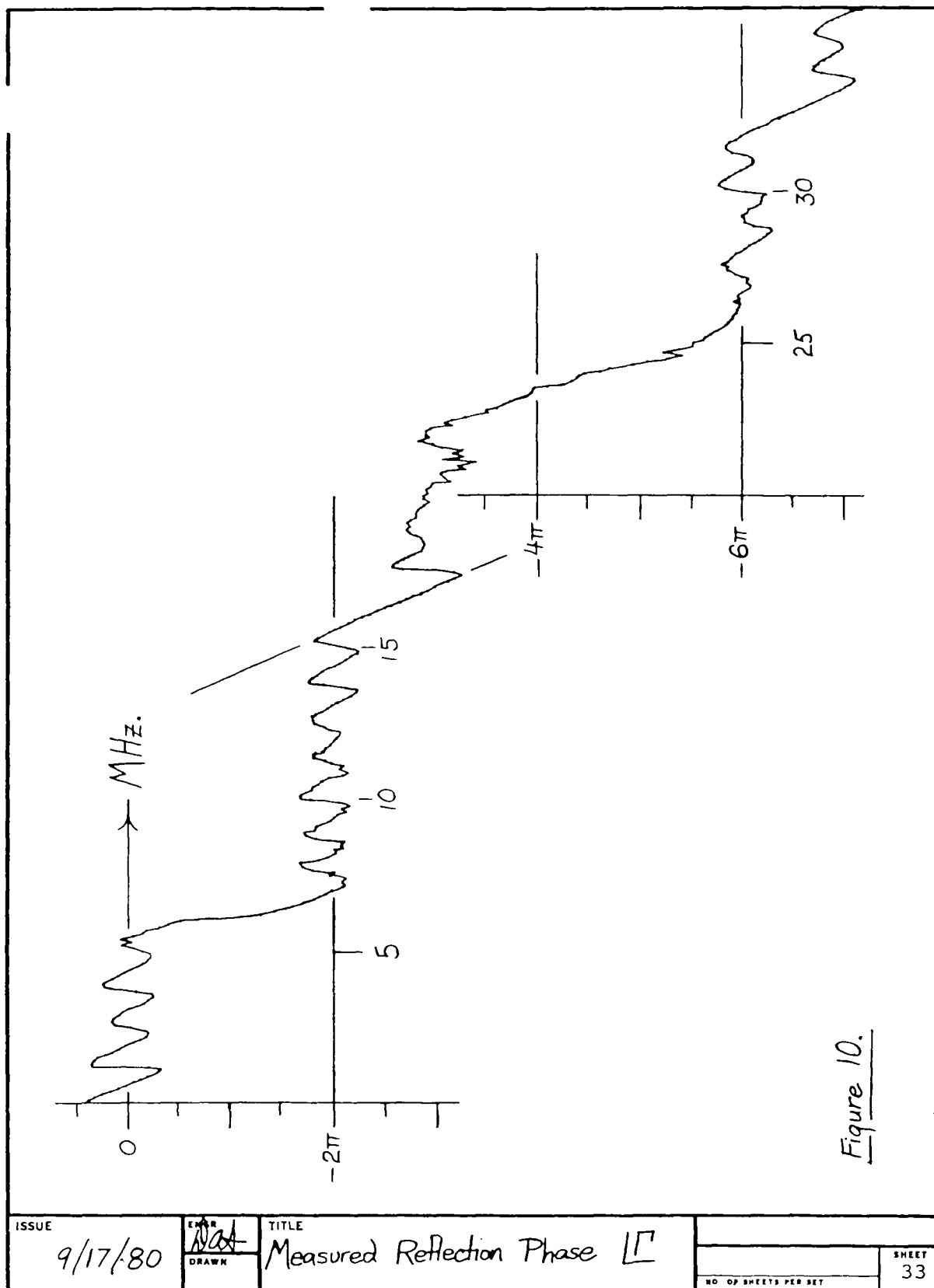
agreeing with the statement that the reflection appears to originate halfway down the test cable.

Notice that the measured phase in the second stopband is $-\frac{\pi}{2}$ and not $+\frac{\pi}{2}$ as predicted. This discrepancy results from the use of small reflection theory to model the unit cell, eq. 4.1. The computer program accounts for all secondary reflections thereby calculating the correct

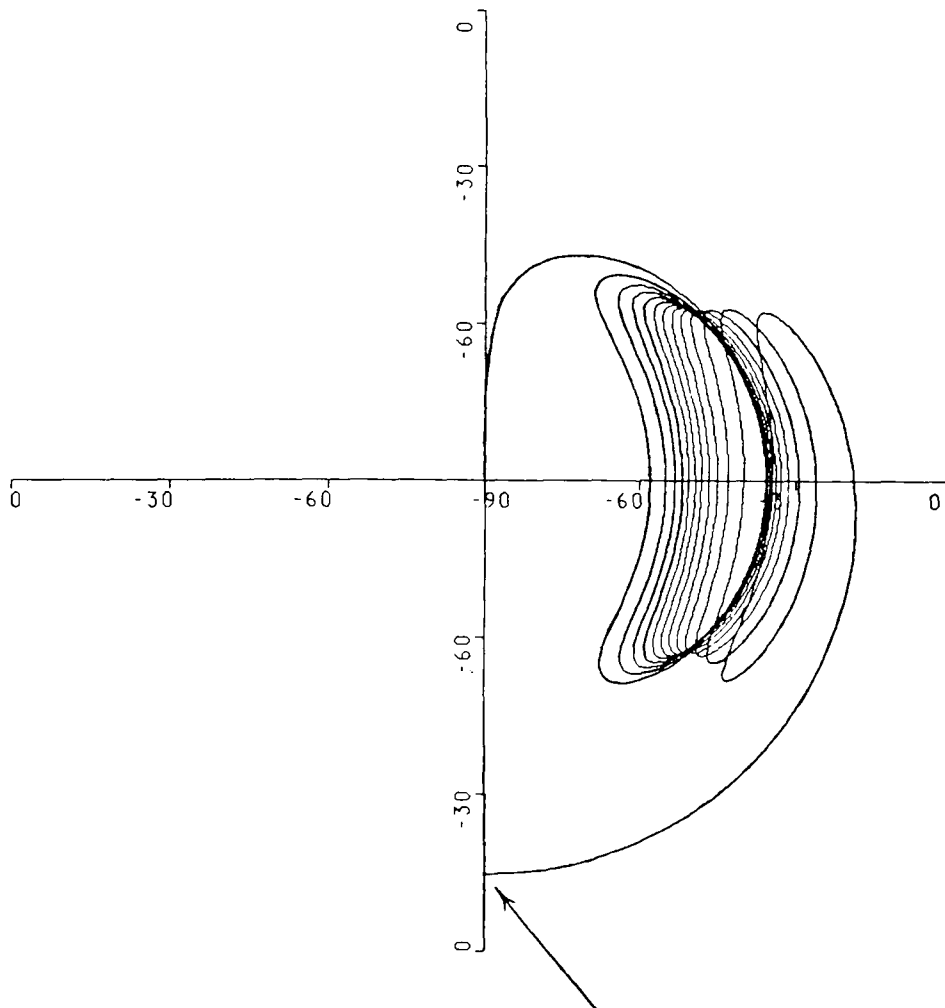
| qth | Frequency | $\Delta\theta$ [radians] |
|----------------------------|-----------|--------------------------------------|
| Minimum | f [MHz] | $\frac{2\pi fd}{c} - \frac{q\pi}{N}$ |
| 1 | 1.20 ± .1 | − .0050 |
| 2 | 2.50 | + .0052 |
| 3 | 3.70 | − .0045 |
| 4 | 5.05 | + .0147 |
| 5 | 6.10 | − .0241 |
| 6 | 7.50 | + .0049 |
| 7 | 8.65 | − .0145 |
| 8 | 10.00 | − .0041 |
| 9 | 11.25 | + .0032 |
| 10 | 12.40 | − .0167 = 1° |
| 11 | 13.75 | + .0019 |
| 12 | 15.10 | + .0210 |
| mth Stopband Maximum | | |
| 1 | 16.25 | + .0003 |
| 2 | 32.65 | + .0233 |

Table 1

Individual Splice Phase Deviation $\Delta\theta$



| | | | | | | | |
|-------|---------|----------|-----|-------|------------------------------------|-----------------------|-------|
| ISSUE | 9/17/80 | ENGINEER | WAT | TITLE | Measured Reflection Phase Γ | NO. OF SHEETS PER SET | SHEET |
| | | DRAWN | | | | | 33 |



center of 1st stopband
radial scale in deciBels with -90 taken as origin
effect of attenuation included

Figure 11.

| | | | |
|-------------------|----------------------|----------------------------------------------------|-------------------------------------|
| ISSUE 11/18/80 | ENGR DRAWN HAA | TITLE Calculated Nyquist Plot for Test Cable | SHEET 34 NO OF SHEETS PER SET |
|-------------------|----------------------|----------------------------------------------------|-------------------------------------|

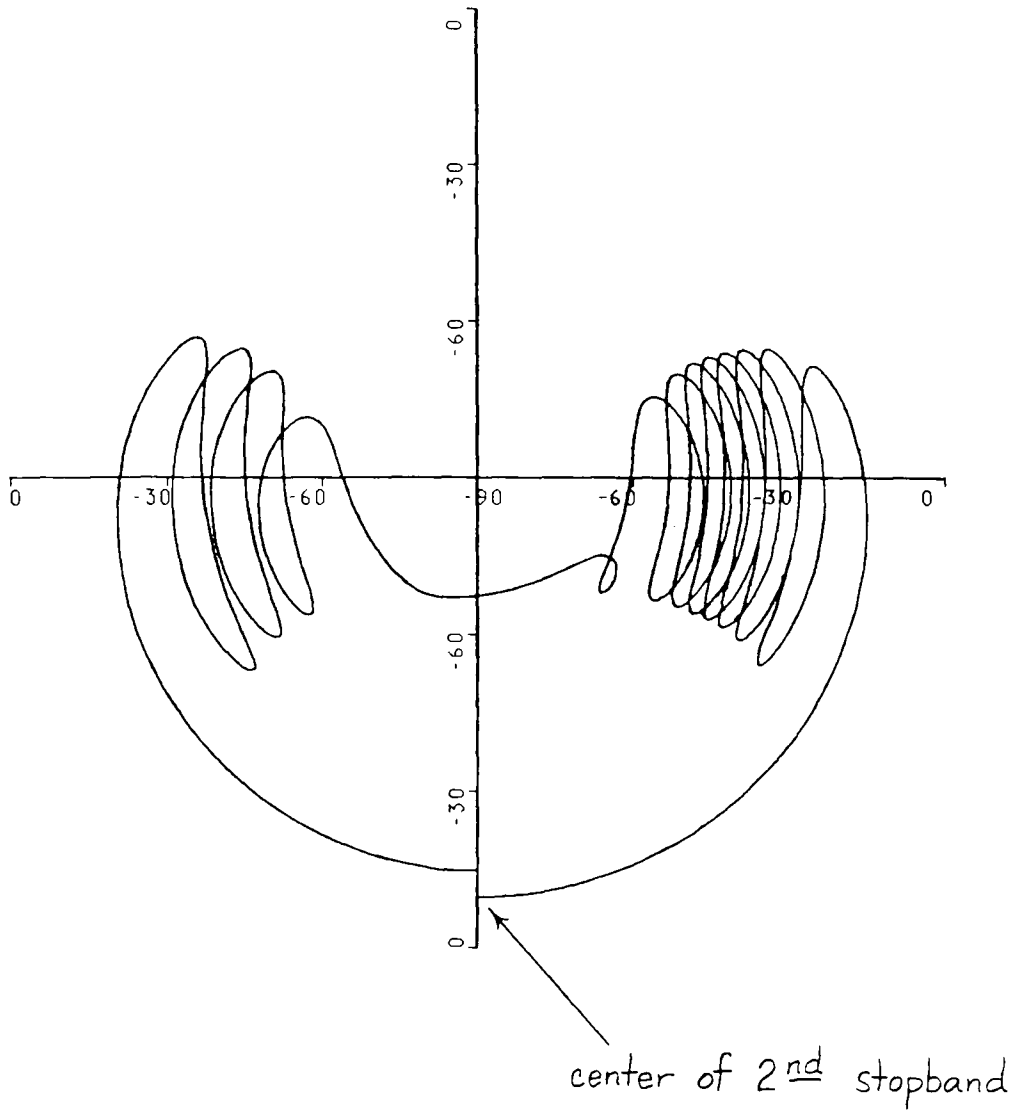


Figure 11. cont.

| | | | |
|-------------------|-------------------------------------|----------------------------------------------------|--------------------------------------|
| ISSUE 11/18/80 | ENGR <i>[Signature]</i> DRAWN | TITLE Calculated Nyquist Plot for Test Cable | NO. OF SHEETS PER SET SHEET 35 |
|-------------------|-------------------------------------|----------------------------------------------------|--------------------------------------|

phase $-\frac{\pi}{2}$. Actually the difference between the computer and simple analytic models occur at $kd = \frac{4}{3}\pi$ when $\rho = 0$. The reflection magnitude is the same with either calculation

Conclusion

With adjustment of the magnitude parameter, $2 \frac{Z-1}{Z+1}$, to account for unmodeled effects the individual splice can be accurately characterized by

$$R = j k l 2 \frac{Z-1}{Z+1}$$

The remaining s matrix elements are obtained by taking the splice to be reciprocal, lossless, and symmetric.

This information may then be used to obtain the reflection and transmission of an arrangement of multiple splices in a uniform line by

- 1) using the analytic formulas for periodic arrangements or
- 2) using the computer program for arbitrary arrangements.

Attenuation in the line is accounted for by reducing the reflection magnitude by the single pass uniform line loss. The transmission magnitude deviates from its nominal value the same amount with or without attenuation.

The excellent agreement between the computer results and the analytic approximations, and between theory and experiment creates confidence when applying the physical model and computer program to arbitrary arrangements of splices.

References

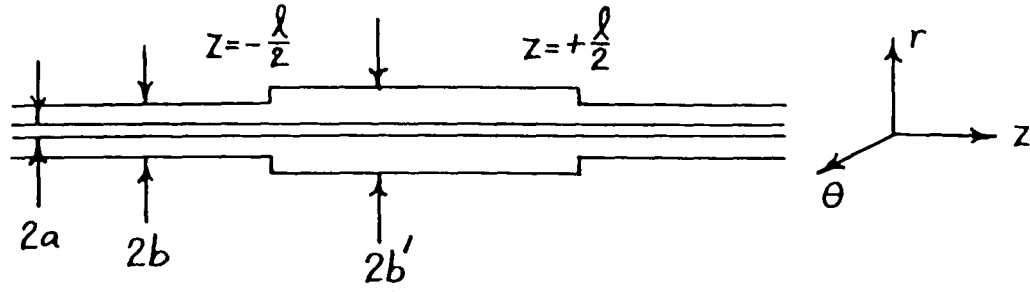
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Appendix 1

Variational Method for the Splice Discontinuity

In Section 1 the splice reflection was computed by modeling the outer conductor diameter change as a simple characteristic impedance change. Here the field equations for the splice 'cavity' will be set up so that the transition boundary conditions can be applied. The equations will be solved by the variational method introduced by Schwinger, ref. 4. While many examples of the technique's application may be found in the Waveguide Handbook, ref. 5, this particular problem is not treated. The notation used here and the method of treating finite length discontinuities parallels that of Collin, ref. 6; however the formulae must be rederived because of differences in the choice of field expansions. The final low frequency result is essentially the same as that of Section 1.

Here again the tapered diameter transition is modeled as an abrupt change at $z = \pm l/2$



Now the electric field $E_r(r)$ at the aperture $z = -l/2$ is a superposition of the forward traveling TEM wave, the reflected wave and the backward traveling higher order modes of the uniform line region

$$E_r(r) = a_0 \phi_0 + a_0 R \phi_0 + \sum_1^{\infty} a_n \phi_n \quad (\text{A1.1})$$

where the $\phi_n(r)$ are the normalized mode generating functions. Since the electric field of the incident wave is radially directed and since the discontinuity is axially symmetric then only higher order E modes need be considered. Also by knowing the wave admittance Y_n of each mode the magnetic field is determined

$$H_\phi(r) = a_0 Y_0 \phi_0 - a_0 Y_0 R \phi_0 - \sum_n a_n Y_n \phi_n \quad (A1.2)$$

Now the aperture fields may also be expressed as a superposition of modes in the splice region. In order to incorporate the splice length l into the expression, the symmetry around $z = 0$ is exploited. Two cases are considered; symmetric excitation of both ports which results in no magnetic field at $z = 0$ and antisymmetric excitation which results in no electric field at $z = 0$. From the reflection coefficients so determined the reflection with one sided excitation may be found.

For antisymmetric excitation the electric field at $z = -l/2$, must have the *form*

$$E_r(r) = \sum_0^\infty b_n \psi_n \left(e^{\frac{\gamma'_n l}{2}} - e^{-\frac{\gamma'_n l}{2}} \right) \quad (A1.3)$$

where ψ_n are the E mode functions for the splice region and γ'_n the associated propagation constants. Then the magnetic field is

$$H_\theta(r) = \sum_0^\infty b_n Y'_n \psi_n \left(e^{\frac{\gamma'_n l}{2}} + e^{-\frac{\gamma'_n l}{2}} \right) \quad (A1.4)$$

with Y'_n the wave admittances for the splice region. Continuity of the fields across the aperture gives

$$a_0 \phi_0 (1 + R_{SC}) + \sum_1^\infty a_n \phi_n = \sum_0^\infty b_n \psi_n \sinh \gamma'_n l/2 \quad (A1.5)$$

$$a_0 Y_0 \phi_0 (1 - R_{SC}) - \sum_1^\infty a_n Y_n \phi_n = \sum_0^\infty b_n Y'_n \psi_n \cosh \gamma'_n l/2 \quad (A1.6)$$

where R_{SC} denotes the reflection for antisymmetric excitation which is equivalent to 'short circuiting' the midplane of the splice. In theory these equations could be solved for the a_n , b_n and R_{SC} ; however in practice approximations of arbitrary precision are conveniently obtained by the calculus of variations.

One assumes the aperture field $E(r)$ is known. Then by multiplying eq. A1.1 by ϕ_m and integrating over the aperture obtain

$$a_0(1+R_{SC}) = \int_a^b E\phi_0 r dr$$

$$a_n = \int_a^b E\phi_n r dr$$

since the ϕ_n are orthogonal functions

$$\int \phi_n \phi_m r dr = \delta_{mn} = \begin{cases} 1, & m=n \\ 0, & m \neq n \end{cases}$$

Similarly from eq. A1.3

$$b_n = \frac{\int_a^b E\psi_n r dr}{\sinh \gamma_n' l/2}$$

Substituting these expressions for the a_n and b_n into eq. A1.6 gives the identity

$$\begin{aligned} \bar{Y}_{SC} Y_0 \phi_0 \int_a^b E \phi_0 r dr = \\ \sum_1^{\infty} Y_n \phi_n \int_a^b E \phi_n r dr + \sum_0^{\infty} Y_n' \psi_n \coth \frac{\gamma_n' l}{2} \int_a^b E \psi_n r dr \end{aligned} \quad (A1.7)$$

where \bar{Y}_{SC} is the normalized input admittance

$$\bar{Y}_{SC} = \frac{1-R_{SC}}{1+R_{SC}}$$

Now the above expression remains true when multiplied by an arbitrary function; in particular choosing $E(r')$ and then integrating over the aperture

$$\begin{aligned} \bar{Y}_{SC} Y_0 \left[\int_a^b E \phi_0 r dr \right]^2 = \\ \int_a^b \int \left[\sum_1^{\infty} Y_n \phi_n(r') \phi_n(r) + \sum_0^{\infty} Y_n' \psi_n(r') \psi_n(r) \coth \frac{\gamma_n' l}{2} \right] E(r') E(r) r' r dr' dr \end{aligned}$$

This is the variational expression for \bar{Y}_{SC} ; for small changes in the assumed form of the aperture field the input admittance will be stationary. To wit if $E + \delta E$ is substituted for E and $\bar{Y}_{SC} + \delta \bar{Y}_{SC}$ for \bar{Y}_{SC} and second order terms neglected then

$$\delta \bar{Y}_{SC} Y_0 \left[\int_a^b E \phi_n r dr \right]^2 = 0$$

where eq. A1.7 multiplied by the arbitrary function δE has been used. Therefore $\delta \bar{Y}_{SC} = 0$.

Now in eq. A1.8 substitute a $N+1$ term approximate expression for E in terms of the uniform line mode functions

$$E = \sum_0^N a_n \phi_n$$

exchange the order of integration and summation and perform the integration using the definition

$$P_{ni} = \int_a^b \psi_n \phi_i r dr \quad (A1.9)$$

to obtain

$$\bar{Y}_{SC} a_0^2 = \sum_{i=0}^N \sum_{j=0}^N a_i a_j q_{ij} \quad (A1.10)$$

where

$$g_{ij} = \sum_{n=1}^{\infty} \frac{Y_n}{Y_0} \delta_{ni} \delta_{nj} + \sum_{n=0}^{\infty} \frac{Y'_n}{Y_0} P_{ni} P_{nj} \coth \delta_n / z \quad (A1.11)$$

Since \bar{Y}_{SC} is constant for small variations in the a_n then differentiating eq. A1.10 with respect to each coefficient yields

$$\begin{cases} 0 = \sum_{i=0}^N a_i g_{i0} - a_0 \bar{Y}_{SC} \\ 0 = \sum_{i=0}^N a_i g_{ij}, \quad j=1, 2, \dots, N \end{cases}$$

This set of $N+1$ homogeneous equations in $N+1$ unknowns has a solution if the determinant vanishes. For instance using a two term approximation and computing the determinant gives

$$\bar{Y}_{sc} = g_{00} - \frac{g_{10}^2}{g_{11}} \quad (A1.12)$$

In a similar fashion the input admittance with symmetric excitation, \bar{Y}_{oc} , is

$$\bar{Y}_{oc} = h_{00} - \frac{h_{10}^2}{h_{11}} \quad (A1.13)$$

where

$$h_{ij} = \sum_{n=1}^{\infty} \frac{Y_n}{Y_0} \delta_{ni} \delta_{nj} + \sum_{n=0}^{\infty} \frac{Y'_n}{Y_0} P_{ni} P_{nj} \tanh \gamma'_n l / 2 \quad (A1.14)$$

Eqs. A1.9, 11-14 may now be evaluated for the splice discontinuity in question. For waveguides in general

$$\gamma_n^2 = k_{cn}^2 - k_0^2$$

where k_{cn} is the cutoff propagation constant for the n th mode. The TEM wave has no cutoff, therefore

$$\gamma_0 = jk_0$$

For the higher order E modes in a coaxial line the k_{cn} are determined by

$$J_0(k_{cn}a)N_0(k_{cn}b) - J_0(k_{cn}b)N_0(k_{cn}a) = 0$$

To within a few percent the roots of this equation are

$$k_{cn} = \frac{n\pi}{b-a}$$

If the maximum frequency considered is 100 MHz then $k_{0,max} = 3.1/m$ while for $2b' = .76cm$, and $2a' = .16cm$ the lowest cutoff is $k'_{c1} = 10^3/m$. Clearly $\gamma_n = k_{cn}$, $n = 1, 2, \dots$. The hyperbolic functions become

$$\tanh \frac{\gamma_0 l}{2} = j \tan \frac{k_0 l}{2} \sim j \frac{k_0 l}{2}, \quad k_0 l \ll 1$$

$$\coth \frac{\gamma_0 l}{2} \sim \frac{2}{jk_0 l}$$

$$\tanh \frac{\gamma_n l}{2} \sim 1 \quad \text{since } \gamma_n l \gg 1, \quad n = 1, 2, \dots$$

and

$$\coth \frac{\gamma_n l}{2} \sim 1$$

For E modes in general

$$\gamma_n Y_n = \gamma_0 Y_0$$

therefore

$$\frac{Y_n}{Y_0} = j \frac{k_0}{k_{cn}}, \quad n = 1, 2, \dots$$

Substituting into A1.11

$$g_{00} = \frac{2}{jk_0 l} \left[P_{00}^2 - \frac{k_1}{2} \frac{k_0}{k_{c1}'} P_{10}^2 + \dots \right]$$

$$g_{10} = \frac{jk_0}{k_{c1}'} P_{11} P_{10} + \dots$$

$$g_{11} = j \left[\frac{k_0}{k_{c1}} + \frac{k_0}{k_{c1}'} P_{11}^2 + \dots \right]$$

Because the frequencies of interest are well below the cutoff for the higher order modes then $k_0 \ll k_{cn}$ and k_{cn}' . Furthermore all the P_{ij} will be of the same order, consequently the only significant contribution to the short circuit admittance is the first term of g_{00} . Eq. A1.12 becomes

$$\bar{Y}_{SC} = \frac{2}{jk_0 l} P_{00}^2 \quad (\text{A1.15})$$

A similar evaluation of the h_{ij} and eq. A1.14 yields

$$\bar{Y}_{oc} = \frac{jk_0 d}{2} P_{00}^2 \quad (A1.16)$$

Now P_{00} is the integral of the product of the normalized TEM mode function for each region. If instead unnormalized functions are used then

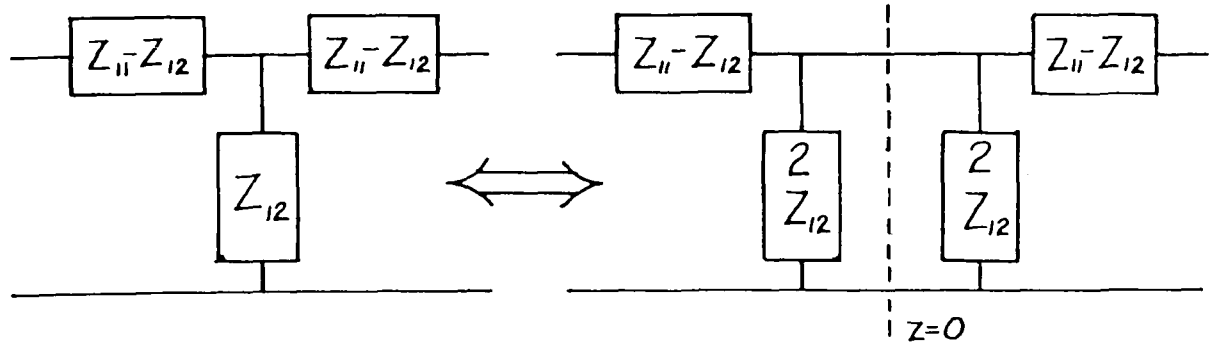
$$P_{00} = \frac{\int_a^b \phi_0 \psi_0 r dr}{\left[\int_a^b \phi_0^2 r dr \int_a^{b'} \psi_0^2 r dr \right]^{\frac{1}{2}}}$$

The TEM fields are inversely proportional to the radius therefore

$$P_{00} = \sqrt{\frac{\ln(b/a)}{\ln(b'/a)}}$$

which can be identified with the normalized splice impedance defined in Section 1, $Z = 1/P_{00}^2$.

To compute the reflection coefficient consider the equivalent T network for the splice



Now antisymmetric excitation corresponds to short circuiting the $z = 0$ plane, therefore

$$\bar{Y}_{sc} = \frac{1}{Z_{11} - Z_{12}}$$

while symmetric excitation gives

$$\bar{Y}_{oc} = \frac{1}{Z_{11} + Z_{12}}$$

The reflection coefficient for the network when terminated into uniform line is

$$R = \frac{\left[Z_{11} - 1 \right] \left[Z_{11} + 1 \right] + Z_{12}^2}{\left[Z_{11} + 1 \right] \left[Z_{11} + 1 \right] + Z_{12}^2}$$

or when expressed in terms of the calculated admittances

$$R = \frac{1 - \bar{Y}_{SC} \bar{Y}_{OC}}{1 + \bar{Y}_{SC} \bar{Y}_{OC} + \bar{Y}_{SC} + \bar{Y}_{OC}}$$

Since $k_0 l \ll 1$ then inspection of A1.15, 16 for \bar{Y}_{SC} and \bar{Y}_{OC} indicates that \bar{Y}_{SC} dominates in the denominator giving

$$R = jk_0 l \frac{1}{2} \left[\frac{1}{P_{00}^2} - P_{00}^2 \right]$$

or

$$R = jk_0 l \frac{1}{2} \left[Z - \frac{1}{Z} \right].$$

Appendix 2

Effect of Attenuation

The effect of attenuation on the observed reflection from periodic discontinuities is determined by taking $\gamma_0 = \alpha_0 + jk$ as the propagation constant of the uniform line. The unit cell scattering parameters become

$$s_{11} = e^{-\gamma_0 d} j\rho$$

$$s_{21} = e^{-\gamma_0 d} \sqrt{1-\rho^2}$$

which when substituted in the eigenvalue eq. 3.1 yield

$$\cosh \gamma d = \frac{a_{11} + a_{22}}{2} = \frac{\cosh \gamma_0 d}{\sqrt{1-\rho^2}}$$

where $\gamma = \alpha + j\beta$ is the Bloch wave propagation constant. A similar calculation gives

$$\frac{a_{11} - a_{22}}{2} = \frac{\sinh \gamma_0 d}{\sqrt{1-\rho^2}}$$

Then the propagation factor and Bloch reflection coefficient are

$$e^{\pm \gamma d} = \frac{\cosh \gamma_0 d \pm \sqrt{\sinh^2 \gamma_0 d + \rho^2}}{\sqrt{1-\rho^2}} \quad (\text{A3.1})$$

$$\Gamma_B^{\pm 1} = \frac{j\rho}{\sinh \gamma_0 d \pm \sqrt{\sinh^2 \gamma_0 d + \rho^2}} \quad (\text{A3.2})$$

If $\alpha_0 d$ is assumed to be much smaller than 1, the hyperbolic functions simplify to

$$\cosh (\alpha_0 + jk)d \sim \cos kd + j\alpha_0 d \sin kd,$$

$$\sinh (\alpha_0 + jk)d \sim \alpha_0 d \cos kd + j \sin kd, \quad |\alpha_0 d| \ll 1$$

Consider first the stopband centers where $kd = m\pi$ and the above become

$$e^{\pm \gamma d} = \frac{1 \pm \sqrt{(\alpha_0 d)^2 + \rho^2}}{\sqrt{1-\rho^2}}$$

$$\Gamma_B^{\pm 1} = \frac{j\rho}{\alpha_0 d \pm \sqrt{(\alpha_0 d)^2 + \rho^2}}$$

Then the reflection, eq. 3.4, becomes

$$\Gamma \sim \pm jN\rho \left\{ 1 - \alpha_0 Nd \right\}, \quad |N\rho| \ll 1, |\alpha_0 Nd| \ll 1$$

So the reflection is less than that expected for lossless lines, eq. 3.15, by the factor

$$1 - \alpha_0 Nd \sim e^{-\alpha_0 Nd}, \quad |\alpha_0 Nd| \ll 1$$

which is just the insertion loss of the reference cable. This is the round trip attenuation of a reflection that appears to originate halfway down the periodic structure.

In the passbands the result is similar. To first order the magnitude of eq. A3.1 is

$$\left| e^{\pm \gamma d} \right| \sim 1 \pm \alpha_0 d, \quad |\alpha_0 d| \ll 1, |\rho| \ll |\sin kd|$$

The phase is affected only in the second order so the dispersion relation is simply

$$\beta = k$$

The Bloch reflection coefficient, eq. A3.2 simplifies to

$$\Gamma_B = \frac{j\rho}{2\alpha_0 d \cos kd + 2j \sin kd}$$

Substituting into eq. 3.11 obtain the reflection for frequencies well within the passband

$$\Gamma = \frac{\rho}{2 \sin kd} \left[1 - j \frac{\alpha_0 d}{\tan kd} \right] \left[1 - \left(1 - 2\alpha_0 Nd \right) e^{2jkNd} \right]$$

The maxima occur when $kNd = \pi \left(q - \frac{1}{2} \right)$, then

$$\Gamma = \frac{\rho}{\sin kd} \left(1 - \alpha_0 Nd \right)$$

where the correction to Γ_B may be neglected. Once again the reflection is reduced by the insertion loss of the reference cable.

Finally when $kNd = q\pi$ the reflection has a local minimum

$$I = \frac{I_0}{\sin kd} = \alpha_0 Nd$$

Therefore the definition, or depth, of the minima is limited to

$$\begin{aligned} -20\log \alpha_0 Nd &= 15\text{dB at } 8\text{MHz} \\ &\text{or } = 10\text{dB at } 24\text{MHz.} \end{aligned}$$

using Fig. 9b; the reference cable attenuation.

Vita

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